

**Example 1**

Let  $k(x) = \frac{x^2}{x-1}$ .

(a) Write the expression

$$\frac{k(a+h) - k(a)}{h}$$

in terms of  $a$  in unsimplified form.

(b) Simplify the expression in part (a) above to show that

$$\frac{k(a+h) - k(a)}{h} = \frac{a^2 - 2a + ah - h}{(a+h-1)(a-1)}$$

(c) Find and simplify the expression

$$\frac{k(a+h) - k(a-h)}{2h}$$

**Example 2**

Let  $f(x) = \log_2 x$ . Show that

$$\frac{f(x+h) - f(x)}{h} = \log_2 \left(1 + \frac{h}{x}\right)^{1/h}$$

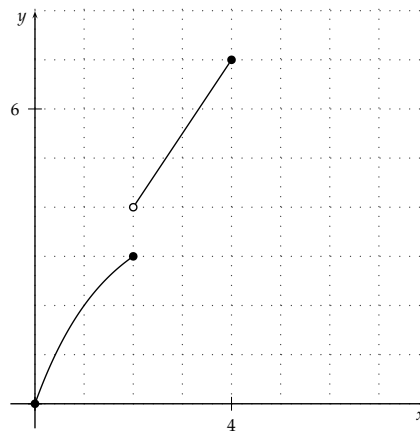
**Example 3**

The graph of a function  $f$  is shown and a partial formula for  $f$  is given below.

$$f(x) = \begin{cases} 4(1 - 2^{-x}) & \text{if } 0 \leq x < 2 \\ \text{_____} & \text{if } 2 \leq x \leq 4 \end{cases}$$

- (a) Give the domain and range of  $f$ , expressed as intervals.
- (b) Complete the formula for the function  $f$ .
- (c) Graph the function  $f^{-1}(x)$ , the inverse of the function  $f$  whose graph is shown above. Give the domain of the inverse.
- (d) Give a formula for  $f^{-1}$  by filling in the following piecewise definition:

$$f^{-1}(x) = \begin{cases} \text{_____} & \text{if } \text{_____} \leq x < \text{_____} \\ \text{_____} & \text{if } \text{_____} \leq x \leq \text{_____} \end{cases}$$



**Example 4**

Simplify the expression:  $\tan(\arcsin x)$ . Hint: Recall that  $\arcsin x = \sin^{-1} x$ .

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**Example 5**

(a) Use the identity

$$\cos(x + y) = \cos x \cos y - \sin x \sin y$$

to prove that

$$\cos 3x = \cos x \cos 2x - \sin x \sin 2x$$

(b) Use the identities

$$\sin 2x = 2 \sin x \cos x, \quad \cos 2x = \cos^2 x - \sin^2 x, \quad \& \quad \cos^2 x + \sin^2 x = 1$$

to prove that

$$\cos 3x = \cos x (1 - 4 \sin^2 x)$$

(c) In the diagram side  $AC = 12$  and side  $CD = 5$ . Give the value of the angle  $\theta$ , in degrees, and use the identity in part (b) above to find the length of the segment  $BD$ .

